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Investigation of mesh independence in numerical modelling of sharp-crested weirs at different heights

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Abstract
Mesh independence is a term used in numerical analysis and simulation to refer to the situation where the numerical solution of a problem is insensitive to changes in the size or density of the computational mesh used to discretize the problem domain. In other words, a numerical solution is said to be mesh independent if it remains stable and accurate even when the mesh is refined. This is important because the accuracy and stability of numerical simulations depend on the quality of the mesh used to discretize the problem domain. The shape, size and number of meshes used while performing numerical modeling in accordance with flow physics in the solution of hydraulic problems significantly affect the result. In this study, a total of 12 experiments were carried out for 4 different discharge values (Q=5 L/s, 10 L/s,15 L/s and 20 L/s) for sharp-crested weirs with three different weir heights (P=20 cm, 30 cm and 40 cm). Numerical models of each experimental setup with 4 different mesh sizes (m=1.25 mm, 2.50 mm, 5.00 mm and 10.00 mm) were created with the ANSYS-Fluent program. Numerical analyzes were performed using the grid resolution method to obtain a mesh size-independent result where the mesh size would not affect the results after the specified error rate.

Keywords: *sharp-crested weirs, numerical modelling, mesh independency*

INTRODUCTION

Structural studies on the use of water started with the existence of humanity because water is an important natural resource for all living things. Water structures are hydraulic structures that take water under control and enable it to be used for various purposes [1]. Weirs are hydraulic structures placed perpendicular to the axis of the open channel and passing water over them to measure the discharge of water, control the water flow and raise the water level. Weirs are generally categorized under two type, broad-crested and sharp-crested. Weirs are one of the most important hydraulic structures that have been used for years for purposes such as flow control, flow measurement, water surface profile managing and changing the flow direction [2]. There are different types and shapes of weirs according to their purpose of usage. The hydraulic behavior and characteristics of weirs of different shapes are also different from each other. In order to design these structures that interacting with the flow, the flow profile, velocity and pressure values must be determined correctly [3]. To achieve mesh independency, the mesh grid system covering the hydraulic problem in the numerical model must be sufficiently fine and detailed. If the mesh grid system is not detailed enough, analysis results may be inaccurate or important details may be missed. Conversely, if the mesh size is too small, the analysis time is prolonged. When making efficient mesh independency investigation, the analysis time and the accuracy of the results should be in a certain balance.

The finite element method is a method used to analyze complex structures by transforming them into mathematical models. In this method, the hydraulic problem is represented by a grid system consisting of meshes (finite elements) [4]. This grid system can usually consist of simple geometric shapes such as triangles, quadrilaterals, trapezoids, or more complex prisms. Mesh independency is a term used in Computational Fluid Dynamics (CFD). This term refers to the representation of fine details of a mesh structure, such as the finite element method used in a simulation or analysis, to obtain accurate results. Mesh independence is important to evaluate the effect of mesh (mesh) structure on results in finite element analysis. To obtain mesh independent results in an analysis, the results of the analysis must be independent of the mesh structure. This indicates that the analysis is more reliable and accurate. A process called "convergence study" is usually followed to ensure mesh independence. In this process, analyzes are made using different mesh densities and the results are compared. As the mesh density increases, the results converge and become invariant at some point. This point is considered the mesh independent solution.

MATERIALS AND METHODS

Sharp Crested Weirs

Sharp-crested weirs are important engineering structures for water management and transportation. These structures are used to raise the water level and control the flow from canals, rivers and other water sources [5]. The upstream water level can be adjusted depending on the weir height [6]. Sharp-edged weirs provide important protection for the environment by keeping the water level under control during heavy rainfall or flooding. The first studies of sharp-edged weirs were made by Boileau [7], Horton [8] and Istomina [9]. Above the sharp-edged weirs located along the cross-section, the flow characteristics change and pass from the river regime to the flood regime [10]. The profile formed when the flow passes freely over a sharp-edged weir is called nappe flow (Fig. 1). In the nappe flow, after the water is passes over the weir, there is air at between wall and water. Sharp-crested weirs are a type of overflow weir [11].

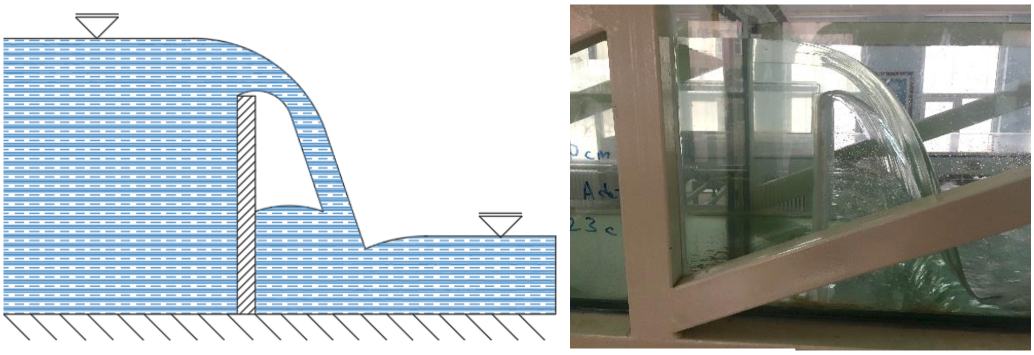


Fig. 1. Nappe flow over a sharp-crested weir

Aeration of the flow over the sharp-crested weirs are defined in 4 different types [12]. These are unventilated-adherent flow (a), partially ventilated flow with chimney (b), self-ventilated and nappe flow state (c), and submerged (suffocated) flow (d) as shown in Fig. 2.

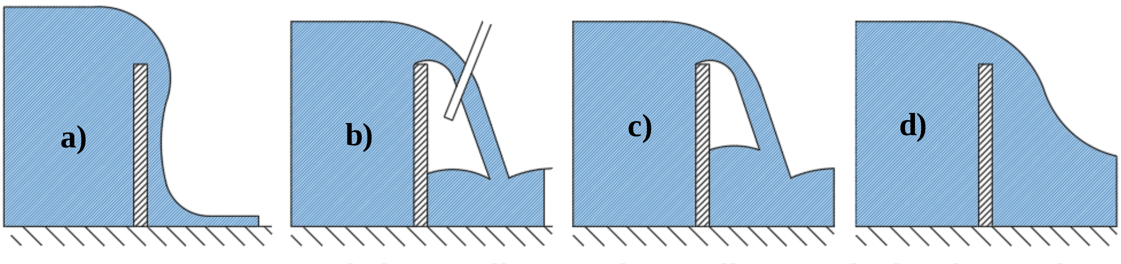


Fig. 2. (a) Non-aerated, (b) Partially-aerated, (c) Fully-aerated, (d) Submerged

Equation (1) for the flow rate passing over a sharp-edged weir is calculated by hand [13]. In this equation, it is assumed that H_T is the total water load and $H_T = h + V^2/2 \times g$.

$$Q = 2/3 * C_d * L_{net} * \sqrt{2 * g} * H_T^{1,5} \qquad (1)$$

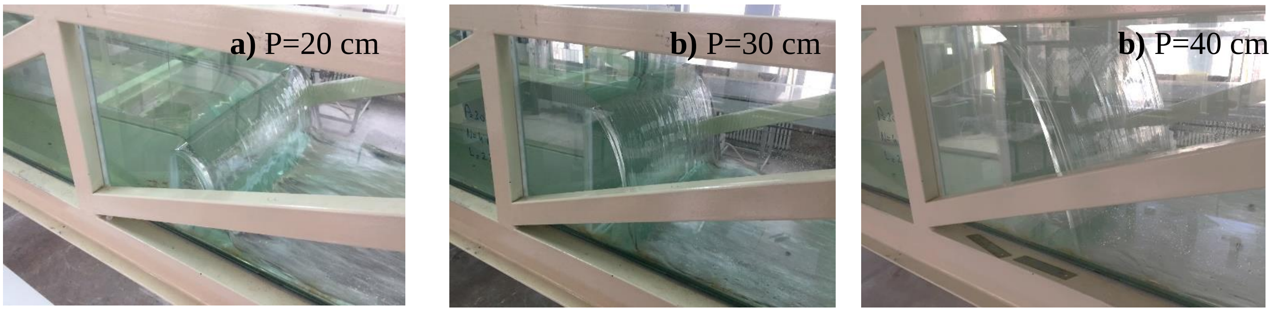
where Q is the discharge, C_d is the discharge coefficient, L_{net} is the net crest length, g is the gravity, H_T is the total head over the weir.

Physical Experiment

The open channel system used in the experiments for the mesh independency test of sharp-crested weirs is shown in Fig. 3. The open channel setup used in the experiments is 6.50 m long, 0.60 m wide and 0.50 m high. Sharp- crested weirs are placed in the 0.3 m part of the channel from the beginning. The purpose of choosing this location is to minimize the fluctuations in the water coming from the reservoir.

Fig. 3. Open channel system used in experiments.

The open channel system works with the circulation of water between the two chambers. The flow rate range given to the system varies between 0.001 and 0.0045 m^3/s. The flow through the system is measured by an ultrasonic flowmeter with a sensitivity of 0.01 L/s placed between the pipes after the pumps. The discharge (Q) and total heads (H_T) passing over the weirs placed in the channel were with a limnimeter. The top of the sharp- crested weirs used in the experiments has a flat crest shape. Since there is no roundness at the top of the weirs, the thickness of the crest is equal to the wall thickness of the weirs. The sharp-crested weirs used in the experiments were made of plexiglass (acrylic) sheets. For sharp-edged weirs, 3 weir heights (P=0.20, 0.30 m and 0.40 m) were used and three set tests were prepared (Fig. 4). 10 mm thick plexiglass was used in the manufacture of the weirs. In the experiments carried out in sharp-edged weirs, attention was paid to the formation of nappe flow, and all experimental data were obtained in the case of nappe flow. No data was received after the aeration under the nap flow was finished and the flow switched to the cohesive flow state. When the images obtained from the experiments are examined, it is observed that self-ventilated nappe flow occurs in all weirs.



a) P=20 cm **b)** P=30 cm **b)** P=40 cm

Fig. 4. Sharp-crested weirs used in experiments.

The results obtained from the sharp-crested weirs are shown in Table 1. Although the total head (H_T) values for only 4 discharge values (Q) are given in Table 1, the experiments carried out for larger discharge range. Since these discharge (Q) values are used in the numerical model, only 5 L/s, 10 L/s, 15 L/s and 20 L/s flow rates and their total head (H_T) values are given.

Table 1. Experimental results of sharp crested weirs

P=20 cm			P=30 cm			P=40 cm		
	Q (L/sn)	H(cm)		Q (L/sn)	H(cm)		Q (L/sn)	H(cm)
1	5.00	2.35	1	5.00	2.44	1	5.00	2.50
2	10.00	3.95	2	10.00	4.00	2	10.00	4.05
3	15.00	5.22	3	15.00	5.26	3	15.00	5.30
4	20.00	6.20	4	20.00	6.30	4	20.00	6.40

Numerical Modelling

CFD (Computational Fluid Dynamics) is a software package for numerical analysis of fluids. CFD is an engineering discipline in which mathematical and numerical methods are used to model and simulate the behavior of fluids (liquids, gases or mixtures). Fluent is a powerful tool for analyzing and optimizing the movement and interaction of fluids in real-world applications. CFD is based on Navier-Stokes equations, which is a mathematical modeling of fluid motion [14]. These equations describe the momentum, mass, and energy of the fluid. The movement of the fluid is calculated by the numerical solution of these equations. In CFD, a grid system is created to calculate the continuous fluid area. This grid is divided into meshes to describe the movement of the fluid. The properties of the fluid are calculated at these cells (meshes) and updated in time or spatial steps. Equations used for solving differential equation sets derived from momentum, energy and mass conservation laws;

The equation for conservation of mass, or continuity equation, can be written as Formulae 2:

$$\partial\rho/\partial t + \nabla * (\rho\vec{v}) = S_m \qquad (2)$$

where ρ is the density of fluid, t is the time, ∇ is the gradient operator, $\mathbf{v}$ is the velocity.

For 2D axisymmetric geometries, the continuity equation is given by Formulae 3

$$\partial\rho/\partial t + \partial/\partial x(\rho v_x) + \partial/\partial r(\rho v_r) + \rho v_r/r = S_m \qquad (3)$$

where ρ is the density of fluid, x is the axial coordinate, t is the time, v_x is the axial velocity, v_r is the radial velocity, r is the radial coordinate.

Conservation of momentum in an inertial (non-accelerating) reference frame is described by Formulae 4

$$\partial/\partial t(\rho\,\vec{v}) + \nabla * (\rho\vec{v}\,\vec{v}) = -\nabla P + \nabla * (\vec{\tau}) + \rho\,\vec{g} + \vec{F} \qquad (4)$$

where ρ is the density of fluid, $\mathbf{v}$ is the velocity, ∇ is the gradient operator, P is the static pressure, g is the gravity, $\vec{F}$ is the body force and $\vec{\tau}$ is the stress tensor.

In the numerical modelling, the environmental conditions of the physical model must be entered into the program correctly in order to obtain the closest results to the experiments. Since the sharp-crested weirs used in the physical experiment setup in 3 dimensions, that is, the same along the channel width, and there are no irregularities. Therefore, numerical models were created in two dimensions. The main purpose of this study is to test the mesh independency with the *Grid resolution* method. If a 3-D numerical model is used, the number of meshes for 60 cm channel width will increase a lot and analysis times will be very long. Numerical models were created in 2-D in the same size as the physical experimental setups and no scaling was made. Flow volume is the volume outside solid surfaces where water and air can be found. Therefore, the size of the flow volume should be determined according to the user's foresight to cover all hydraulic events. The sizes and mesh numbers of the meshes used are shown in the Table 2.

Table 2. Mesh size and numbers used for numerical modelling sharp-crested weirs

Model	Mesh Size (mm)	Grid Size (cm x cm)	Total Mesh Number
P=20 cm	1.25	35x160	359360
	2.50	35x160	89840
	5.00	35x160	22460
	10.00	35x160	5615
P=30 cm	1.25	45x160	461760
	2.50	45x160	115440
	5.00	45x160	28860
	10.00	45x160	7215
P=40 cm	1.25	55x160	564160
	2.50	55x160	141040
	5.00	55x160	35260
	10.00	55x160	8815

In the analysis of the sharp-crested weir, the flow volume was determined depending on the weir height. The flow volumes to be used in the analysis were created in AutoCAD and transferred to the program in .sat (Standard ACIS Text) format (Figure 4.52). The dimensions of the flow volume used in the analysis are shown in the table. Its height and length were chosen so that the flow and spillage of water over circular weirs remains within the analysis. Continuity (Navier-Stokes) equation is solved in each mesh (cell). As the number of cells contained in the flow volume increases, the accuracy of the solution also increases. The geometry of the flow volume of the numerical model is a rectangular prism, the shape of the meshes is also chosen as a cube. Cartesian mesh was used (Fig. 5).

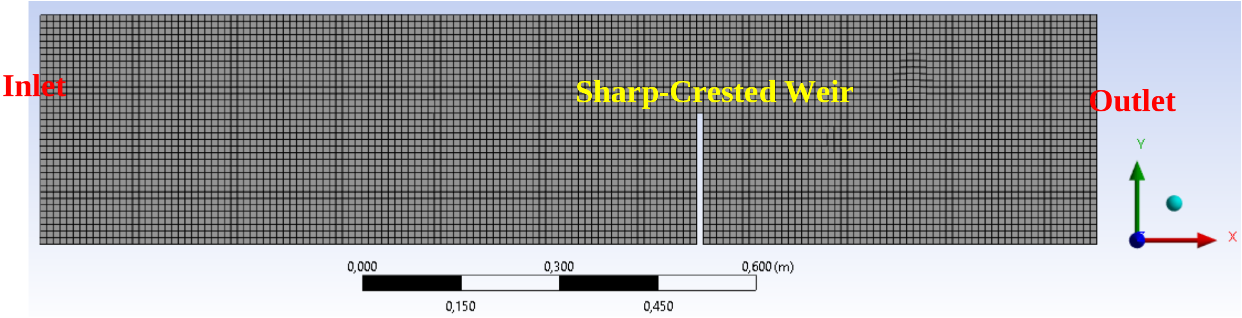


Fig 5. Boundary conditions and cartesian meshes used in numerical model.

The “inlet” part of the flow volume is defined as the “mass flow inlet”. Mass flow inlet is determined in kg/s, that is, L/s. The “outlet” part of the flow volume is defined as the “pressure outlet”. Air and water can come out freely from this layer (Fig. 5). The k-omega RNG was used as the turbulence model. In experiments carried out in the laboratory, there is a free surface water stream and air above it. Therefore, in the analysis, it was designed as a 2-phase so that both air and water are in the flow volume. The “Multiphase” model was chosen as the VOF (Volume of Fluid) method to provide the two-phase current. The effect of air on water (open air pressure) is also considered in this way. To make the solution more detailed, an "explicit" solution was used instead of an "implicit" solution. In the analysis, a solution was made according to the variable flow (Transient Flow). To shorten the analysis time, a water volume is defined behind the weirs. In this way, the analysis time spent to fill the back of the weirs with water is avoided. In the analysis, “time step size” varies between 0.001 and 0.005 seconds depending on the height of the weirs, flow rate and mesh size.

Mesh independency test methods

There are three commonly used mesh independence methods.

Grid resolution (Mesh refinement)

Based on prior knowledge of the problem to be solved with the numerical model, reducing, or increasing the mesh size is one of the commonly used ways of making mesh-independent solutions. In the Grid Resolution method, the mesh size is determined based on prior knowledge of the physics of the problem. The mesh number is gradually decreased until there is no significant performance increase in solving the problem [15]. In other words, if there is no significant change in the results even though the mesh size is reduced, the optimum mesh size is determined [16].

General Richardson Extrapolation (GRE)

The Richardson method uses an expansion series to determine exact values. Convergence and improvement rates are used to minimize the margin of error [17]. This expansion series is a series of performance prediction variable, grid improvement rate, and order of accuracy. In this method, at least three mesh sizes are required to calculate the order of accuracy [18].

Grid Convergence Index (GCI)

The Grid Convergence Index (GCI) method was derived from the General Richardson Extrapolation method by Roache [19]. As in the Richardson Extrapolation method, 3 different mesh sizes are needed to estimate the extrapolated values and the accuracy estimation value is also needed [20]. Relates the error from systematic mesh reductions or enlargements to the error produced when doubling or halving the mesh size using the quadratic method.

RESULTS AND DISCUSSION

3 different weir heights (P=20 cm, P=30 cm and P=40 cm) and 4 different discharge values (Q=5 L/s, Q=10 L/s, Q=15 L/s and Q=20 L/s) are used on sharp-crested weirs, at total of 12 different experiments were carried out. A total of 48 numerical models were created for each of these experimental sets in 4 different mesh sizes (s=10 mm, s=5 mm, s=2.5 mm and s=1.25 mm). In this study, the aim is to find the mesh size that will provide the optimum consistency between the experimental results and the numerical model, while creating the numerical model of a sharp-crested weir, considering the analysis times.

The results obtained from the numerical models are shown in Table 3. Total head values over the sharp-crested weir obtained from the numerical models and the error values calculated based on their comparison with the experimental results are given. When the error values are examined, it is seen that the error value decreases as the mesh sizes get smaller and the mesh number increases. For all weir heights (P=20 cm, P=30 cm and P=40 cm), the percentage avg. error for the 10 mm mesh size is 12%, the mean error for the 5 mm mesh size is 7%, the mean error for the 2.5 mm mesh size is 3% and 1.25%. The average error for mm mesh size is 1%.

Table 3. Numerical results of shar-crested weirs for different mesh sizes

Mesh size=1.25 mm											
P=20 cm				P=30 cm				P=40 cm			
	Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)
1	5.00	2.32	0.01	5	5.00	2.39	0.02	9	5.00	2.48	0.01
2	10.00	3.90	0.01	6	10.00	3.96	0.01	10	10.00	4.01	0.01
3	15.00	5.13	0.02	7	15.00	5.15	0.02	11	15.00	5.19	0.02
4	20.00	6.10	0.02	8	20.00	6.24	0.01	12	20.00	6.34	0.01
	Avg. Error		0.01		Avg. Error		0.02		Avg. Error		0.01
Mesh size=2.50 mm											
P=20 cm				P=30 cm				P=40 cm			
	Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)
13	5.00	2.23	0.05	17	5.00	2.37	0.03	21	5.00	2.40	0.04

14	10.00	3.85	0.03	18	10.00	3.88	0.03	22	10.00	3.93	0.03
15	15.00	5.00	0.04	19	15.00	5.05	0.04	23	15.00	5.04	0.05
16	20.00	6.00	0.03	20	20.00	6.17	0.02	24	20.00	6.21	0.03
	Avg. Error		0.04		Avg. Error		0.03		Avg. Error		0.04
Mesh size=5.00 mm											
	P=20 cm				P=30 cm				P=40 cm		
	Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)
25	5.00	2.15	0.09	29	5.00	2.24	0.08	33	5.00	2.35	0.06
26	10.00	3.67	0.07	30	10.00	3.72	0.07	34	10.00	3.73	0.08
27	15.00	4.93	0.06	31	15.00	4.79	0.09	35	15.00	4.88	0.08
28	20.00	5.80	0.06	32	20.00	5.92	0.06	36	20.00	5.95	0.07
	Avg. Error		0.07		Avg. Error		0.08		Avg. Error		0.07
Mesh size=10.00 mm											
	P=20 cm				P=30 cm				P=40 cm		
	Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)		Q (L/s)	H(cm)	E (%)
37	5.00	2.20	0.06	41	5.00	2.10	0.14	45	5.00	2.18	0.13
38	10.00	3.65	0.08	42	10.00	3.52	0.12	46	10.00	3.60	0.11
39	15.00	4.62	0.11	43	15.00	4.63	0.12	47	15.00	4.66	0.12
40	20.00	5.40	0.13	44	20.00	5.48	0.13	48	20.00	5.63	0.12
	Avg. Error		0.10		Avg. Error		0.13		Avg. Error		0.12

There is no rule in the literature about what exactly the error should be for consistency between the numerical model and the experimental study. What should be the consistency of the results obtained is entirely related to the physics and importance of the hydraulic problem. In this study, it will be sufficient for the results of the numerical models of the sharp-edged weirs to be 5% consistent with the experimental models. Because the precision of the limnimeter used when measuring the total head over the sharp-crested weirs is 1 mm and this value comes up to 10% of cm. There may already be a margin of error of 10% from the errors to be made during the measurement. In addition, when measuring with a limnimeter, the water sticks to the tip of the limnimeter a little and causes an error in the situation. Analyzes with 1.25 mm and 2.5 mm mesh sizes gave less than 5% avg error. It would be better to use the results obtained with a mesh size of 1.25 mm with a percentage avg. error of 1%, but analysis times should also be considered here. In order for a numerical model to be an efficient model, its results should be below the desired error rates, consistent with the experimental data, and the analysis times should not be too long according to the severity of the problem. While the analysis with a mesh size of 2.5 mm took 3 hours to conclude, the model with a mesh size of 1.25 mm took 9 hours to conclude.

The main reason why the amount of error decreases as the mesh sizes get smaller and the result is closer to the experimental data is due to the fact that the solution in a certain region is done with more finite elements. For example, when Fig. 6 is examined, there is only one mesh in the weir width in the solution made with 10 mm, while this number increases to 8 when the solution is made with 1.25 mm. In addition, as the mesh number increases, the visual resolution of the results also increases.

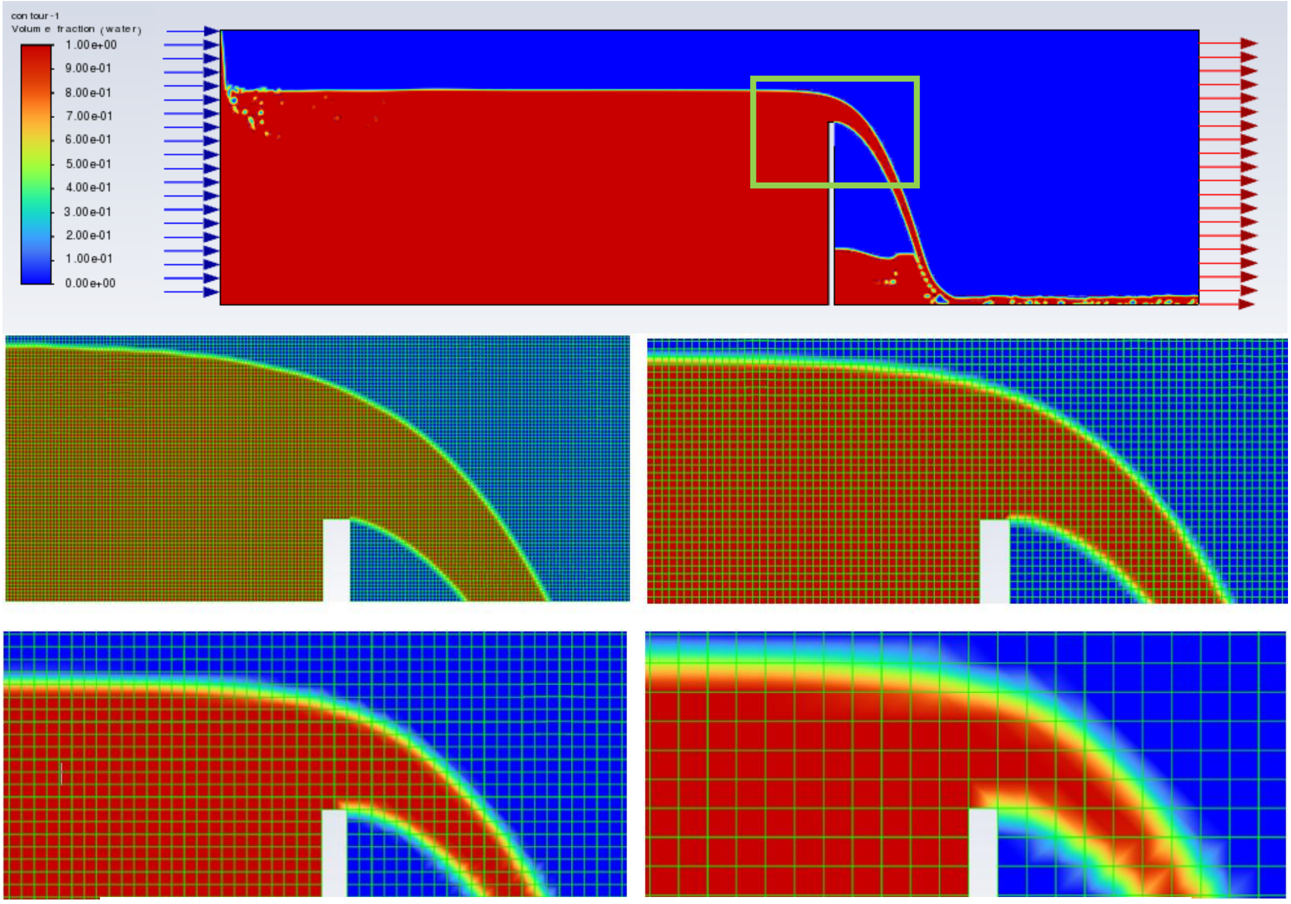


Fig 6. 2D result images of numerical models made with mesh sizes of different sizes.

CONCLUSIONS

In this study, mesh independency research was carried out for numerical modeling of a sharp-sharp crested weirs. Different mesh sizes have been used, which decreased at a certain rate. It was tried to determine the mesh size that gave the most consistent result between the numerical model and the experimental study, with the error rate below the determined value. While the most consistent numerical model was used in terms of results, attention was paid to be efficient in terms of analysis time. When the 2D images obtained from the numerical model are compared with the experiments, it is seen that they are extremely similar in terms of water surface profiles and flow characteristics. While the water flows freely over the weirs, the flow condition is Partially-aerated on the downstream side and the nappe flow is clearly visible.

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REFERENCES

[1] KUMAR, P.M., HALDER, P., SAMAD, A., RHEE, S.H., Int. J. Fluid Mach. Syst., **11**, no. 1, 2018, p. 55, https://doi.org/10.5293/IJFMS.2018.11.1.055.

[2] ACKERS, P., WHİTE, W.R., PERKINS, J.A., AND HARRISON, A.J.M., Weirs and flumes for flow measurement, Wiley, New York, 1978.

[3] NOVAK, P., MOFFAT, A.I.B., NALLURRI, C., NARAYANAN, R., Hydraulic Structures, Taylor and Francis, London, 2007.

[4] FERZIGER, J. H., VE PERIC, M., Computational Methods for Fluid Dynamics, Springer, New York, 1999.

[5] BOS, M.G., Discharge measurement structures, International Institute for Land Reclamation and Improvement (ILRI) Publication, 1998.

[6] BAGHERI, S., HEIDARPOUR, M., Irrig. Sci., **28**, no. 1, 2010, p. 173, https://doi.org/10.1007/s00271-009-0172-1.
[7] BOILEAU, P., Traite de la mesure des eaux courantes, Mallet-Bachelier, Prag, 1854.
[8] HORTON, R.E., Weir experiments, coefficients, and formulas, Government printing office, Washington, 1907.
[9] ISTOMINA, V.S., L'Energia Elettrica, **14**, no. 2, 1937.
[10] HENDERSON F.M., Open-channel flow, Macmillan, New York, 1989, p. 269.
[11] ESCANDE, L., SABATHE, G., Comptes Rendus de l'Académie des Sciences, **204**, 1937, p. 1547.
[12] FALVEY, H. T., Hydraulic design of labyrinth weirs, American Society of Civil Engineers (ASCE), Virginia, 2002.
[13] AYDIN, İ., ALTAN, S., B., ŞİŞMAN, C., Flow Meas. Instrum., **22**, no. 2, 2011, p. 144, https://doi.org/10.1016/j.flowmeasinst.2011.01.003.
[14] MOUKALLED, F., MAGANI, L., DARWISH, M., The Finite Volume Method in Computational Fluid Dynamics, Springer International Publishing, Switzerland, 2016.
[15] ALMOHAMMADİ, K.M., INGHAM, D.B., MA, L., POURKASHAN, M., Energy, **58**, 2019, p. 483, https://doi.org/10.1016/j.energy.2013.06.012.
[16] KUMAR S., AHMAD Z., MANSOOR T., J. Flow Meas. Instrum., **22,** no. 1, 2011, p. 175.
[17] RICHARDSON L.F., Philos. Trans. Royal Soc. A, **210**, no. 459, 1911, p. 307.
[18] ROY, C.J., J. Comput. Phys., **205**, no. 1, 2005, p. 131, https://doi.org/10.1016/j.jcp.2004.10.036.
[19] ROACHE P., J. Fluids Eng. Sep., **116**, no. 3, 1994, p.405, https://doi.org/10.1115/1.2910291.
[20] CELIK, I.B., GHIA, U., ROACHE, P.J., FREITAS, C.J., COLEMAN, H., RAAD, P.E., J. Fluids Eng., **130**, no. 7, 2008, https://doi.org/10.1115/1.2960953.

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